CRITICAL DESIGN REVIEW

Analytical Calculations and Dynamic Models

Deployment Team Report

November 2016
Issue no. 1
## Changes

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Abbreviated terms

ADCS  Attitude Determination and Control System
AP    Argument of Perigee
AR    Acceptance Review
COMM  Communication subsystem
DT    Deployment Team
EM    Engineering Model
EPS   Electrical Power System
ESA   European Space Agency
FM    Flight Model
FRR   Flight Readiness Review
GS    Ground Station
IADC  Inter-agency space debris coordination committee
LEO   Low Earth Orbit
MA    Mission Analysis
MDR   Mission Definition Review
PDR   Preliminary Design Review
SC    Spacecraft
SKA   Studenckie Koło Astronautyczne (Students’ Space Association)
SW    Software
TBC   To Be Continued
TBD   To Be Defined
TCS   Thermal Control System
WUT   Warsaw University of Technology
1 INTRODUCTION

1.1 PURPOSE AND SCOPE

Purpose of this document is to share analytical calculations and dynamic models of mechanisms on board satellite PW-Sat2.

1.2 DOCUMENT STRUCTURE

Chapter 1 introduces the document, documentation structure, reference documents and main objectives of this very overview.

Chapter 2 contains the calculations related to the conical spring.

Chapter 3 summarizes SRM static analysis.

Chapter 4 presents calculations of torque spring in SADS.

Chapter 5 is a summary of this document.

1.3 PROJECT DOCUMENTATION STRUCTURE

See section 1.3 in [PW-Sat2-C-00.00-Overview-CDR].

1.4 REFERENCE DOCUMENTS


1.5 APPLICABLE PROJECT DOCUMENTS

- [PW-Sat2-C-00.00-Overview-CDR] – the overview of the PW-Sat2 Phase C
- [PW-Sat2-C-05.00-DT-CDR] – the overview of the PW-Sat2 Deployment Team
- [PW-Sat2-C-05.01-DT-Structural-Analysis]
- [PW-Sat2-C-10.00-CONF-CDR] – Configuration overview of PW-Sat2
- [PW-Sat2-C-10.01-CONF-MICD] – Mechanical Interface Control Document describing DT structures
- [PW-Sat2-C-10.02-CONF-MICD-Drawing] – Mechanical Interface Control Document Technical Drawing which includes a detailed view on the satellite
• [PW-Sat2-C-10.03-COMF-Bill-of-Materials] (spreadsheet) – Bill of Materials (spreadsheet)
• [PW-Sat2-C-11.01-Tests-Plan-Mechanical] – document describing mechanical tests of the satellite systems
• [PW-Sat2-C-11.02-Tests-Plan-Thermal] – document describing thermal tests of the satellite systems
• Assembly plans of the DT structures (see section 1.3 in [PW-Sat2-C-00.00-Overview-CDR])

1.6 DOCUMENT CONTRIBUTORS

This document and any results described were prepared solely by PW-Sat2 project team members.
2 CONICAL SPRING CALCULATION

2.1 PURPOSE OF CALCULATIONS

Purpose of calculations is to compare analytical model of conical spring which with Finite Element Method model. The conical spring is used in Sail Release Mechanism, within this spring sail will be deployed above the satellite. (read more in [PW-Sat2-C-05.00-DT-CDR], chapters 2.1., 2.2.) The FEM calculations were performed because the analytical methods are not accurate enough for this setup.

2.2 SPRING DESCRIPTION

Dimensions of conical spring used in analysis shows Figure 1. Number of active coils is 8. Assumed material constants are: Young’s modulus $E= 200$ GPa, Poisson’s ratio $\nu=0,3$ and are the same for both cases. Spring wire diameter is 1,4 mm.

![Figure 2-1 Conical spring dimensions](image)

2.3 FEM ANALYSIS

Model used during FEM analysis depicts Figure 2. Flat circular plate simulates container and is fixed on the bottom (point A). End of spring (point B) is also fixed. Spring was gradually deflecting (point C) and after that, force required to deflect was calculated. Figure 3 shows fully compressed conical spring.
Figure 2-2 Model of the spring during FEM analysis

Figure 2-3 Fully compressed spring
2.4 COMPARISON OF METHODS

Analytical solution was performed using formulas from [1]. Figure 2-4 FEM and analytical characteristics of conical spring shows compared both methods.

![Comparison of FEM and Analytical Characteristics](image)

**Figure 2-4 FEM and analytical characteristics of conical spring**

Analytical and Finite Element Method solutions are almost identical. Difference is no more than 10% and probably is caused by simplifications in analytical model and differences in geometry. This proves that FEM is very accurate in conical spring simulation and will allow simulate springs with variable pitch, different endings and non-circular spring wire.
3 SRM STATICS ANALYSIS

Author: Maksymilian Gawin

3.1 OVERVIEW

The purpose of that part is to present the set of calculations that was done on SRM (to read more about this subsystem, see [PW-Sat2-C-05.00-DT-CDR], chapter 2.3.) to compare results the calculation was done in two ways: analytical calculation and numerical model in ADAMS software. After getting the same results from both methods proper information about forces that act on mechanism and its behavior is known.

3.2 ANALYTICAL MODEL

3.2.1 MODEL DESCRIPTION

The most important thing before creating an analytical model is to identify forces the way they affect the mechanism. When all active forces are known the second step is to figure out the passive forces.

![Activ and passiv forces acting on SRM](image)

Figure 3-1 Activ and passiv forces acting on SRM
Active forces are $F_{CS}$ which is conical spring force, and forces $F_{SL}$ and $F_{SR}$ - springs forces. The springs was chosen from the Spring catalogue and values of their forces are fixed. The rest of marked forces are resultant forces.

### 3.2.2 Results

Using denotation shown above and mechanical equations values of all passive forces was obtained as functions of conical spring force, springs forces and dimensions $a$, $b$, $c$, $d$.

$$F_{CS} = R_{1ZL} + R_{1ZL}, \quad R_{1ZL} = R_{1ZL}$$

$$F_{CS} = 2R_{1ZL}$$

$$R_{1ZL} = R_{1ZL} = \frac{1}{2} F_{CS}$$

$$R_{1YL} = R_{1YR}$$

$$\tan(\alpha) = \frac{R_{1YR}}{R_{1ZL}}$$

$$R_{1YR} = R_{1ZL} \cdot \tan(\alpha), \quad R_{1YL} = R_{1ZL} \cdot \tan(\alpha)$$

\[
\begin{align*}
R_{1YL} \cdot d - F_{SL} \cdot c - R_{2YL} \cdot b + R_{3YL} \cdot a &= 0 \\
R_{1YL} - F_{SL} - R_{2YL} + R_{3YL} &= 0
\end{align*}
\]
\[
\begin{align*}
R_{1YR} \cdot d - F_{SR} \cdot c - R_{2YR} \cdot b + R_{3YR} \cdot a &= 0 \\
R_{1YR} - F_{SR} - R_{2YR} + R_{3YR} &= 0
\end{align*}
\]

The Table 3-1 shows results of calculations done with specific values of active forces, and dimensions a, b, c, d given by actual geometry of mechanism.

**Table 3-1 Results of analytical calculations of SRM**

<table>
<thead>
<tr>
<th>characteristic</th>
<th>denomination</th>
<th>value</th>
<th>unit</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>magnitude of the conical spring's force</td>
<td>F_SC</td>
<td>16</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>magnitude of the springs' force</td>
<td>F_SL = F_SR</td>
<td>4</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>the angle between components of the reaction force on the arbor</td>
<td>A</td>
<td>45</td>
<td>°</td>
<td>fixed values</td>
</tr>
<tr>
<td>dimensions</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>20</td>
<td>mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>25</td>
<td>mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>31.5</td>
<td>mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>46</td>
<td>mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>reaction forces</td>
<td></td>
<td></td>
<td></td>
<td>calculated values</td>
</tr>
<tr>
<td>R_1IZL</td>
<td>8,0</td>
<td>N</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R_1YL</td>
<td>8,0</td>
<td>N</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R_3YL</td>
<td>4,55</td>
<td>N</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R_2YL</td>
<td>8,55</td>
<td>N</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The important point of that calculation was to find out the needed value of force that have to be provided by the flat spring (red colored spring on Figure 3-1 Activ and passiv forces acting on SRM). That value equals to values of $R_{3YR}$.

**Table 3-2 Results of analytical calculations of SRM**

<table>
<thead>
<tr>
<th>characteristic</th>
<th>denomination</th>
<th>value</th>
<th>unit</th>
<th>comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>needed force of the flat spring</td>
<td>F</td>
<td>4,55</td>
<td>N</td>
<td></td>
</tr>
</tbody>
</table>

Then the force F (needed flat spring force) was known it was required to find out information about material, dimensions and expected deflection.

Basic dimensions of the flat spring was established. Values of density and elasticity coefficient was taken as for steel, the material, which was found as the best option for that element.
Table 3-3 Results of analytical calculations of SRM

<table>
<thead>
<tr>
<th>characteristic</th>
<th>denomination</th>
<th>value</th>
<th>unit</th>
<th>comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>length</td>
<td>L</td>
<td>11,00</td>
<td>mm</td>
<td>fixed values</td>
</tr>
<tr>
<td>height</td>
<td>H</td>
<td>0,35</td>
<td>mm</td>
<td></td>
</tr>
<tr>
<td>width</td>
<td>B</td>
<td>4,00</td>
<td>mm</td>
<td></td>
</tr>
<tr>
<td>density</td>
<td>ro</td>
<td>7,86</td>
<td>g/cm^3</td>
<td>fixed values</td>
</tr>
<tr>
<td>elasticity coefficient</td>
<td>E</td>
<td>205000</td>
<td>Mpa</td>
<td></td>
</tr>
</tbody>
</table>

Then using theoretical equation for a beam, deflection of the flat spring impacting by the force \( F \) was calculated from equation

\[
f = \frac{F \cdot l^3}{3E \cdot I_x}
\]

where \( I_x = \frac{b \cdot t^3}{12} \) is moment of inertia.

The result of the calculations are presented in the Table 4.

Table 3-4 Final results of analytical calculations

<table>
<thead>
<tr>
<th>characteristic</th>
<th>denomination</th>
<th>value</th>
<th>unit</th>
<th>comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>moment of inertia</td>
<td>Ix</td>
<td>0,01</td>
<td>mm^4</td>
<td>calculated values</td>
</tr>
<tr>
<td>volume</td>
<td>V</td>
<td>15,40</td>
<td>mm^3</td>
<td></td>
</tr>
<tr>
<td>mass</td>
<td>M</td>
<td>0,12</td>
<td>g</td>
<td></td>
</tr>
<tr>
<td>deflection</td>
<td>F</td>
<td>0,69</td>
<td>mm</td>
<td></td>
</tr>
</tbody>
</table>

The resultant deflection is acceptable in comparison with dimension of the mechanism, the flat spring will be manufactured with dimensions and material properties presented above.

3.3 **ADAMS MODEL**

3.3.1 **MODELS DESCRIPTION**

To confirm correctness of previous calculations the model of SRM in ADAMS software was built. Material properties for every element was assigned. Every joint was modelled as ADAMS’ manuals advice and how they are expected to work.

- Fixed Joint, (removes all degrees of freedom)
- Revolute Joint (removes all three translational DOF (x, y, z) and the two rotational DOF with axes perpendicular to the hinge axis).
Figure 3-3 presents used joints.

Contact between some parts was added (see Figure 3-3). Active forces was applied to the model. Two small springs were even removed from the model and replaced by forces applied in the correct point.

![Figure 3-3 Joints used in ADAMS model](image)

### 3.3.2 SIMULATION AND RESULTS

The simulation was play with Bars fixed in position shown in Figure 3-4 to find out the characteristics of passive forces to get sufficient knowledge in order to compare both methods of calculations. During simulations series of characteristics shown in Figure 3-4 was got.

![Forces](image)
Figure 3-4 Bars used in ADAMS model

Figure 3-5 Characteristic of passive forces in function of conical spring force (vertical axis shows values of reaction forces, horizontal axis shows value of flat spring)
Figure 3-6 Vertical axis shows value of reaction force, horizontal axis shows value of the force on the conical spring (see [2], chapter 2.2.)

When analyzing results of simulations for each force, the same values were obtained, of course numerical values were afflicted by some errors, but small enough to be sure that results from both methods are equal.

Thanks to those simulation sureness of correctness of results was obtained.

Knowing that ADAMS model was correct some dynamic simulation was done to know how the mechanism behave after deployment. The time required to deploy, the way how arbor goes are now known thanks to those simulations.
4 **CALCULATIONS OF SPRING TORQUE FOR SADS**

*Author: Ewelina Ryszawa*

4.1 **INTRODUCTION**

Solar Arrays Deployment System (SADS) is based on the torsion springs. Due to the small room for the whole mechanism it was necessary to use small springs but with required torque (more information about SADS system in: [PW-Sat2-C-05.00-DT-CDR], chapter 2.4.). Calculations are based on the torque budget necessary for safe Solar Panels deployment.

4.2 **CALCULATIONS**

Before choosing proper springs it was necessary to investigate what will be friction torques and resistances during the Solar Arrays movement.

The main friction torque comes from wires which are connecting the EPS with Solar Arrays. Some types of the cables were tested using dynamometer and dummy-hinges. Measurements were made in the temperature around -50degC using dry ice for achieving the cold case. Results from this measurements are shown on the graph below:

![Wires resistance torque](image)

**Figure 4-1 Wires resistance torque in SADS torque spring**

Based on the wires measurements the maximum resistance torque was estimated on 18Nmm.
Friction torque is estimated to be not higher than 4Nmm

Summary of all friction torques for one hinge is maximum 22Nmm – based on the hinges construction it is required that the residual torque must be above 26Nmm (4Nmm above maximum resistance).

It was decided to use torsion springs from Gutekunst Federn catalogue and with 180deg arms’ position. Proposed springs: T16124 and T16224.

Initial measurements showed that holes for mounting arms of the springs must be shifted to achieve proper angles range. Based on this assumptions the required working angles for the springs were chosen (from CAD model of the hinges):

Figure 4-2 Geometry of the spring 16124 in SADS mechanism: in the stowed (left) and deployed position (right)

Figure 4-3 Geometry of the spring 16224 in SADS mechanism: in the stowed (left) and deployed position (right)
In the Table 4-1 calculations of torques for both springs are shown:

Table 4-1 Residual torque calculations for springs T16124 and T16224

<table>
<thead>
<tr>
<th></th>
<th>Spring 16124</th>
<th>Spring 16224</th>
</tr>
</thead>
<tbody>
<tr>
<td>stowed angle</td>
<td>67</td>
<td>55</td>
</tr>
<tr>
<td>deployed angle</td>
<td>157</td>
<td>145</td>
</tr>
<tr>
<td>working angle</td>
<td>113</td>
<td>125</td>
</tr>
<tr>
<td>maximum angle (allowed from manufacturer datasheet)</td>
<td>137</td>
<td>133</td>
</tr>
<tr>
<td>residual angle</td>
<td>23</td>
<td>35</td>
</tr>
<tr>
<td>residual torque</td>
<td>2,59Nmm</td>
<td>7,94Nmm</td>
</tr>
</tbody>
</table>

This calculation shows that spring 16124 cannot be used in the design (too low residual torque, 10,4 Nmm for 4 springs). For spring 16224 the residual torque (for 4 springs) will be ~31,8Nmm which is above the estimated minimum value.

On the graph below the torque budget for each Solar Panel (2 hinges) is shown.

Figure 4-4 Torque budget for Solar Panel
Additionally calculations of the maximum acceleration of the hinge in the area where it hits the structure were made. In the worst case (no friction or wires resistance torque is present in the mechanism) this acceleration will be ~4.4g. In normal case (maximum values of friction and wire's resistance torque) this acceleration will be smaller than 0.1g.
5 SUMMARY

Calculations and analyses described in this paper helped to optimize solutions developed on board PW-Sat2 satellite and prove the effectiveness of created mechanisms. Analytical calculations were compared with numerical simulations to obtain the correctness of results.

Calculations have been consulted with research scientists at WUT, who confirmed that the subsystems were designed the right way.

Results of calculations presented in this document will be also validated with physical tests, e.g. solar panels opening test in temperature -40 Celsius degrees in vacuum chamber.